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Pricing Geometric Asian Power Options under Mixed Fractional Brownian Motion Environment

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Abstract : It has been observed that the stock price process can be modeled with driving force as a mixed fractional Brownian motion with Hurst index $H > \frac{3}{4}$ whenever long-range dependence is possibly present. We obtain a closed form expression for the price of a geometric Asian option under the mixed fractional Brownian motion environment. We consider also Asian power options when the payoff function is a power function.

Keywords and phrases: Mixed fractional Brownian motion; Option price; Asian option; Asian power option.

MSC 2010: 62P05

1 Introduction

Estimation of option price is an important problem in mathematical finance. A call option is a contract which gives the holder the right but not obligation to buy a risky asset at a certain date called the strike date or the exercise date with a predetermined price called the strike price or the exercise price. A put option is a contract which gives the holder the right but not obligation to sell a risky asset at a certain date called the strike date or the exercise datewith a predetermined price called the strike price or exercise price. There are several types of options that are traded in a market. American option allows the owner to exercise his option at any time up to and including the strike date. Bermuda options permit the owner to exercise his option early but only on a contractually specified finite set of dates. European options can be exercised only on the strike date. European options are also called vanilla options. Their payoffs at maturity depend on the spot value of the stock at the time of exercise. There are other options whose values depend on the stock prices over a predetermined time interval. For an Asian option, the payoff is determined by the average value over some predetermined time interval. Asian options reduce the volatility inherent in the option and are cheap compared to the European option (cf. Mao and Liang (2014), Prakasa Rao (2013)). For modelling of fluctuations in movement of stock prices, Brownian motion has been used traditionally as the driving force for modelling log returns. It has been noted later that there might be long-range dependence in the phenomena and the log returns have possibly heavy tailed distibutions. It was suggested by some that the driving force for modelling of price movement may be chosen as a fractional Brownian motion. Bjork and Hult (2005) and Kuznetsov (1999) observed that the use of fractional Brownian motion for modelling fluctuations in movement of stock prices is not justifiable as it allows arbitrage opportunities. To avoid this problem, Cheridito (2000, 2003) suggested the use of a mixed fractional Brownian motion as a suitable model to capture the flutuations of the financial assets. The mixed fractional Brownian motion (mfBm) is a Gaussian process that is linear combination of the Brownian motion and a fractional Brownian motion with Hurst index $H > 1/2$. Cheridito (2001) has proved that, for $H \in (3/4, 1)$, the mfBm is equivalent to a Brownian motion and hence modeling price fluctuation via mfBm allows arbitrage-free market. Xiao et al. (2012) studied pricing model for equity warrants in a mixed fractional Brownian environment. Sun (2013) investigated pricing currency options when the driving force is a mixed fractional Brownian motion. Yu and Yan (2008) discussed European call option pricing under a mixed fractional Brownian motion environment. Mao and Liang (2014) evaluated geometric Asian option under fractional Brownian motion frame work. They derived a closed form for the solution for the Asian power option price. The pricing of currency options in a mixed fractional Brownian motion in a jump environment has been studied in Foad and Adem (2014) and Prakasa Rao (2015). Sun and Yan (2012) discussed use of mixed-fractional models in credit risk pricing. Our aim is to evaluate the price of Asian power options under a mixed fractional Brownian motion environment.

2 Asian Options

The payoff of an Asian option is determined by the average value of the stock price over a pre-fixed time interval as it reduces the risk of market manipulation of the underlying instrument at maturity and reduce the volatility in the option. Furthermore, Asian options are generally cheaper than the corresponding European options. Asian options are of different types such as fixed strike price options and floating strike price options. The payoff for a fixed strike price option is $(A(T)-K)_+$ and $(K-A(T))_+$ for a call and put option respectively where K denotes the strike price, T is the strike time and $A(T)$ is the average price of the underlying asset over the predetermined interval. For a floating strike price option, the payoofs are $(S(T) - A(T))_+$ and $(A(T) - S(T))_+$, for a call and put option respectively where $S(T)$ is the price of stock at time T. Asian options can again be differentiated in to two classes: one is the arithmetic average, that is,

$$A(T) = \frac{1}{T}\int_0^T S(t)dt$$

and the other is the geometric average

$$A(T) = \exp\{\frac{1}{T}\int_0^T \log S(t)dt\}$$

assuming that the pre-fixed interval for computing the average is the interval $[0, T]$. We will consider evaluation of Asian option price in the case of continuous geometric average with a fixed strike price in an mfBm environment.

3 Mixed fractional Brownian Motion

We now define the mixed fractional Brownian motion (mfBm) and discuss some of its properties.

A mixed fractional Brownian motion $M^H(\alpha, \beta)$ is a linear combination of a Brownian motion and a fractional Brownian motion (fBM) with Hurst index H, that is,

$$M_t^H(\alpha, \beta) = \alpha W_t + \beta W_t^H, 0 \leq t < \infty \tag{3. 1}$$

where W is the standard Brownian motion and W^H is an independent standard fractional Brownian motion with Hurst index H and α, β are some real constants not both zero. The equality here is understood in the sense that the finite dimensional distributions of the process on the left side of the equation (3.1) are the same as the corresponding finite dimensional distributions of the process on the right side of the equation (3.1). The process $M^H(\alpha, \beta)$ is a centered Gaussian process with $M_0^H = 0$ a.s. and with the covariance function

$$cov(M_t^H, M_s^H) = \alpha^2 \min(t, s) + \frac{\beta^2}{2}(t^{2H} + s^{2H} - |t - s|^{2H}).$$

The increments of the process $M^H(\alpha, \beta)$ are stationary and self-similar, in the sense that, for any $h > 0$,

$$M_{ht}(\alpha, \beta) \stackrel{\Delta}{=} M_t^H(\alpha h^{1/2}, \beta h^H). \tag{3. 2}$$

Here Δ indicates that the random variables on both sides of the equation (3.2) have the same distribution. The increments of the process are positively correlated if $\frac{1}{2} < H < 1$, uncorrelated if $H = \frac{1}{2}$ and negatively correlated if $0 < H < \frac{1}{2}$. The increments of the process are long-range dependent if and only if $\frac{1}{2} < H < 1$. For more details on the properties of a mfBm, see Zili (2006) and Prakasa Rao (2010). We assume here after that that the index $H > \frac{3}{4}$ which ensures that the probability measure generated by the process $M^H(\alpha, \beta)$ is equivalent to the Wiener measure. For simplicity in computations, we assume that $\alpha = \beta = 1$ here after. Integration of an adapted process with respect to the mixed fractional Brownian motion is defined as the sum of the Ito integral of the adapted process with respect to the Wiener process W and the path-wise integral of the adapted process with respect to the to the fractional Brownian motion W^H. Conditions for the existence and uniqueness of the solution for a stochastic differential equation (SDE) driven by mfBm are given in Section 3.2 in Mishura (2008). However, we will not be using the SDE model for modeling the stock price process in the following discussion.

4 Pricing model

We assume that the following assumptions hold:
(i) the dynamics of the underlying stock price follows the mixed fractional Brownian motion (mfBM) with Hurst index $H > \frac{3}{4}$;
(ii) the risk-free interest rate $r(t)$ is a non-random function;
(iii) there are no transaction costs in buying or selling the stocks or options, that is, the market is frictionless;
(iv)the option can be exercised only at the time of maturity.

The fact that $H > \frac{3}{4}$ ensures that the market does not admit arbitrage opportunity. Suppose the stock price process $\{S_t, t \geq 0\}$ satisfies the model

$$(4.\ 1) \quad S_t = S_0 \exp(g(t) + \sigma W_t + \sigma W_t^H - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}), S_0 = S(0) > 0, 0 \leq t \leq T$$

for some non-random function $g(t)$ where W is the Brownian motion and W^H is an independent fBm with Hurst index $H > \frac{3}{4}$. We will first consider the case when the risk-free interest rate is r and the dividend rate is q. Under the risk neutral probability measure, from the general theory of option pricing and the fact that the risk-neutral measure is a martingale measure (cf. Shreve (2004); Cheridito (2003)), the dynamics of the stock price process $\{S(t), t \geq 0\}$ will satisfy the

$$(4.\ 2) \qquad S_t = S_0 \exp((r-q)t + \sigma(W_t + W_t^H) - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}), 0 \leq t \leq T.$$

This implies that the stock price $S(t)$ is log-normally distributed with

$$\log S(t) \simeq N(\log S(0) + (r-q)t - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}, \sigma^2 t + \sigma^2 t^{2H}).$$

Here $N(m, \nu^2)$ denotes the Gaussian distribution with mean m and variance ν^2. Let $C(S(0),T)$ be the price of a European call option at time 0 with strike price K that matures at time T. Following Theorem 4.1 in Sun (2013), it follows that

$$C(S(0),T) = S(0)e^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \tag{4. 3}$$

where

$$d_1 = \frac{\log(S(0)/K) + ((r-q)T + \frac{\sigma^2}{2}T + \frac{\sigma^2}{2}T^{2H}}{\sqrt{\sigma^2 T + \sigma^2 T^{2H}}}$$
$$d_2 = d_1 - \sqrt{\sigma^2 T + \sigma^2 T^{2H}},$$

and $\Phi(.)$ denotes the standard normal distribution function. Applying the put-call option parity formula (cf. Ross (2003), Prakasa Rao (2013)), it is easy to obtain the option price for a European put option under the above scenario.

5 Pricing for Asian options when the interest and dividend rates are constant

We now obtain a closed form for the price for the geometric Asian call option with fixed strike price K and maturity time T.

Theorem 5.1: Suppose the stock price $S(t)$ follows the model given by the equation (4.2) under the risk-neutral probability measure where the interest rate r and the dividend rate q are constant over time. Then the price of a geometric Asian call option $C(S(0),T)$ is given by

(5. 1)
$$\begin{aligned} C(S(0),T) &= S(0)\exp\{-\frac{1}{2}(r+q)T - \frac{1}{12}\sigma^2 T - \frac{1}{4(2H+1)(H+1)}\sigma^2 T^{2H}\}\Phi(d_1) \\ &\quad -Ke^{-rT}\Phi(d_2) \end{aligned}$$

where

$$d_2 = \frac{\log(S(0)/K) + \frac{1}{2}(r-q)T - \frac{1}{4}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H}}{\sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}} \tag{5. 2}$$

and

$$d_1 = d_2 + \sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}. \tag{5. 3}$$

Proof : Let

$$G(T) = \frac{1}{T}\int_0^T \log S(t)dt$$

and

$$A(T) = \exp(G(T)).$$

From earlier computations, it is clear that the random variable $G(T)$ has Gaussian distribution under the risk-neutral probability measure. We will now compute its mean and variance under the risk-neutral probability measure. Let $\tilde{E}$ denote the expectation and, $\tilde{\mu}$ and $\tilde{\sigma}^2$ denote the mean and the variance of the random variable $G(T)$ under the the risk-neutral probability measure. Note that

$$\begin{aligned} \tilde{\mu} &= \tilde{E}[G(T)] = \frac{1}{T}\int_0^T \tilde{E}[\log S(t)]\, dt \\ &= \log S(0) + \frac{1}{T}\int_0^T (r-q)tdt - \frac{1}{2T}\int_0^T [\sigma^2 t + \sigma^2 t^{2H}]dt \\ &= \log S(0) + \frac{1}{2}(r-q)T - \frac{1}{2}(\frac{\sigma^2 T}{2} + \frac{\sigma^2 T^{2H}}{2H+1}) \end{aligned} \tag{5. 4}$$

and

$$\begin{aligned} \tilde{\sigma}^2 &= Var[G(T)] = \tilde{E}[G(T) - \tilde{\mu}]^2 \\ &= \frac{1}{T^2}\int_0^T\int_0^T \sigma^2(\tilde{E}[W(t)W(\tau)] + \tilde{E}[W^H(t)W^H(\tau)])\, dtd\tau \\ &\qquad \text{(by the independence of the processes } W \text{ and } W^H) \\ &= \frac{1}{2T^2}\int_0^T\int_0^T \sigma^2[(|t| + |\tau| - |t-\tau|) + (|t|^{2H} + |\tau|^{2H} - |t-\tau|^{2H})]\, dtd\tau \\ &= \frac{1}{3}\sigma^2 T + \frac{1}{2(H+1)}\sigma^2 T^{2H}. \end{aligned} \tag{5. 5}$$

Hence the random variable $A(T)$ is log-normally distributed and the random variable $\log A(T)$ has the Gaussian distribution with the mean $\tilde{\mu}$ and the variance $\tilde{\sigma}^2$ as obtained above. For the geometric Asian option, the price of a call option is

$$C(S(0), T) = e^{-rT}\tilde{E}[(A(T) - K)_+] \tag{5. 6}$$

$$= \quad e^{-rT}\int_J (e^x - K)\frac{1}{\sqrt{2\pi}\tilde{\sigma}}\exp\{-\frac{(x-\tilde{\mu})^2}{2\tilde{\sigma}^2}\}\,dx$$

where J is the set $[x : e^x > K]$. The following computations give the explicit formula for the function $C(S(0), T)$. Let $\phi(.)$ denote the probability density function of a standard normal distribution. Observe that

(5. 7)

$$\begin{aligned}
C(S(0),T) &= e^{-rT}\int_J (e^{\tilde{\mu}+\tilde{\sigma}y} - K)\phi(y)dy \\
&= e^{-rT+\tilde{\mu}+\frac{1}{2}\tilde{\sigma}^2}\int_{-d_2}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-\tilde{\sigma})^2}dy \\
&\quad -Ke^{-rT}\int_{-d_2}^{\infty}\phi(y)dy \\
&= e^{-rT+\tilde{\mu}+\frac{1}{2}\tilde{\sigma}^2}\int_{-d_2-\tilde{\sigma}}^{\infty}\phi(y)dy - Ke^{-rT}\Phi(d_2) \\
&= S(0)\exp\{-\frac{1}{2}(r+q)T - \frac{1}{4}\sigma^2 T \\
&\quad +\frac{1}{6}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H} + \frac{1}{4(H+1)}\sigma^2 T^{2H}\}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\
&= S(0)\exp\{-\frac{1}{2}(r+q)T - \frac{1}{12}\sigma^2 T - \frac{1}{4(2H+1)(H+1)}\sigma^2 T^{2H}\}\Phi(d_1) - Ke^{-rT}\Phi(d_2)
\end{aligned}$$

where

(5. 8)
$$\begin{aligned}
J &= \{x : A(T) > K\} = \{y : e^{\tilde{\mu}+\tilde{\sigma}y} > K\} \\
&= \{y : \tilde{\mu} + \tilde{\sigma}y > \log K\} \\
&= \{y : y > -d_2\}
\end{aligned}$$

and d_1 and d_2 are as defined by (5.3) and (5.2) respectively. This completes the proof of Theorem 5.1.

Applying similar arguments, it can be shown that the price $P(S(0), T)$ of a geometric Asian put option under the mfBm environment is given by

(5. 9)

$$P(S(0),T) = Ke^{-rT}\Phi(-d_2) - S(0)\exp\{-\frac{1}{2}(r+q)T - \frac{1}{12}\sigma^2 T - \frac{1}{4(2H+1)(H+1)}\sigma^2 T^{2H}\}\Phi(-d_1)$$

where d_1 and d_2 are as defined by (5.3) and (5.2) respectively.

6 Pricing for Asian power options when the interest and dividend rates are constant

We will now consider computation of the price of Asian Power call option under mfBm environment where the payoff for a call option with strike price K and maturity time T is $(A^n(T) - K)_+$ for some fixed integer $n \geq 1$.

Theorem 6.1: Suppose the stock price $S(t)$ follows the model given by the equation (4.2) under the risk-neutral probability measure where the interest rate r and the dividend rate q are constant over time and the payoff function at the time of maturity is $(A^n(T) - K)_+$. Then the price of geometric Asian power call option $C(S(0), T)$ is given by

(6. 1)

$$\begin{aligned} C(S(0),T) &= S^n(0)\exp\{-rT + \frac{n}{2}(r-q)T - \frac{n}{4}\sigma^2 T + \frac{n^2}{6}\sigma^2 T \\ &\quad - \frac{n}{2(2H+1)}\sigma^2 T^{2H} + \frac{n^2}{4(H+1)}\sigma^2 T^{2H}\}\Phi(D_1) \\ &\quad - Ke^{-rT}\Phi(D_2) \end{aligned}$$

where

(6. 2)
$$D_2 = \frac{\log(S(0)/K^{1/n}) + \frac{1}{2}(r-q)T - \frac{1}{4}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H}}{\sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}}$$

and

(6. 3)
$$D_1 = D_2 + \sqrt{\frac{n^2\sigma^2 T}{3} + \frac{n^2\sigma^2 T^{2H}}{2(H+1)}}.$$

Proof : For the Asian power option, the payoff function is $(A^n(T) - K)_+ = (\exp(nG(T)) - K)_+$. Following the arguments given in Theorem 5.1, it follows that

(6. 4)
$$\begin{aligned} C(S(0),T) &= e^{-rT}\tilde{E}[(A^n(T) - K)_+] \\ &= e^{-rT}\int_J (e^{nx} - K)\frac{1}{\sqrt{2\pi}\tilde{\sigma}}\exp\{-\frac{(x-\tilde{\mu})^2}{2\tilde{\sigma}^2}\}\,dx \end{aligned}$$

where J is the set $[x : e^{nx} > K]$. The following computations give an explicit formula for the function $C(S(0), T)$. Observe that

(6. 5)

$$
\begin{aligned}
C(S(0),T) &= e^{-rT}\int_J (e^{n(\tilde{\mu}+\tilde{\sigma}y)}-K)\phi(y)dy \\
&= e^{-rT+n\tilde{\mu}}\int_{-D_2}^{\infty}\frac{1}{\sqrt{2\pi}}e^{n\tilde{\sigma}y-\frac{y^2}{2}}dy - Ke^{-rT}\int_{-D_2}^{\infty}\phi(y)dy \\
&= e^{-rT+n\tilde{\mu}+\frac{1}{2}n^2\tilde{\sigma}^2}\int_{-D_2}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-n\tilde{\sigma})^2}dy - Ke^{-rT}\Phi(D_2) \\
&= e^{-rT+n\tilde{\mu}+\frac{1}{2}n^2\tilde{\sigma}^2}\Phi(D_1) - Ke^{-rT}\Phi(D_2) \\
&= S^n(0)\exp\{-rT+\frac{n}{2}(r-q)T-\frac{n}{4}\sigma^2T+\frac{n^2}{6}\sigma^2T \\
&\qquad -\frac{n}{2(2H+1)}\sigma^2T^{2H}+\frac{n^2}{4(H+1)}\sigma^2T^{2H}\}\Phi(D_1) \\
&\qquad -Ke^{-rT}\Phi(D_2)
\end{aligned}
$$

where D_1 and D_2 as defined in (6.3) and (6.2) respectively. Note that

(6. 6)
$$
\begin{aligned}
J &= \{x : A^n(T) > K\} = \{y : e^{n(\tilde{\mu}+\tilde{\sigma}y)} > K\} \\
&= \{y : n(\tilde{\mu}+\tilde{\sigma}y) > \log K\} \\
&= \{y : y > -D_2\}.
\end{aligned}
$$

This completes the proof of Theorem 6.1.

In a similar way, the price of an Asian power put option with payoff function $(K-A^n(T))_+$ is

(6. 7)
$$
\begin{aligned}
P(S(0),T) &= Ke^{-rT}\Phi(-D_2) \\
&\qquad -S^n(0)\exp\{-rT+\frac{n}{2}(r-q)T-\frac{n}{4}\sigma^2T+\frac{n^2}{6}\sigma^2T \\
&\qquad -\frac{n}{2(2H+1)}\sigma^2T^{2H}+\frac{n^2}{4(H+1)}\sigma^2T^{2H}\}\Phi(-D_1).
\end{aligned}
$$

7 Price of Asian Power Option under general non-random rate and dividend functions

We will now consider computation of the price of Asian call option under mfBm environment where the payoff for a call option with the strike price K and the maturity time T is $(A^n(T)-K)_+$ and the interest rate and the dividend rate are non-random functions $r(t)$ and $q(t)$ respectively. Under the risk-neutral probability measure, the dynamics of the stock price is obtained by the equation

(7. 1) $S(t) = S(0)\exp(\int_0^t (r(s)-q(s))ds - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H} + \sigma W_t + \sigma W_t^H), 0 \leq t \leq T$

where $W = \{W_t, 0 \leq t \leq T\}$ is the standard Brownian motion and $W^H = \{W_t^H, 0 \leq t \leq T\}$ is an independent standard fBm. Hence the random variable $\log S(t)$ has the Gaussian distribution with mean

$$\log S(0) + \int_0^t (r(s)-q(s))ds - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}$$

and the variance

$$\sigma^2 t + \sigma^2 t^{2H}.$$

Note that

$$A(T) = \exp\{\frac{1}{T}\int_0^T \log S(t)dt\}. \tag{7. 2}$$

It can be checked that the random variable $\log A(T)$ has the Gaussian distribution with mean $\tilde{\mu}$ and variance $\tilde{\sigma}^2$, where

$$\tilde{\mu} = \log S(0) - \frac{1}{4}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H} + \frac{1}{T}\int_0^T \int_0^t (r(s)-q(s))dsdt \tag{7. 3}$$

and

$$\tilde{\sigma}^2 = \frac{1}{3}\sigma^2 T + \frac{1}{2(H+1)}\sigma^2 T^{2H}. \tag{7. 4}$$

Furthermore the price of Asian power call option is given by

$$\begin{aligned} C(S(0),T) &= e^{-\int_0^T r(s)ds}\tilde{E}[(A^n(T)-K)_+] \\ &= e^{-\int_0^T r(s)ds}\int_J [e^{n(\tilde{\mu}+\tilde{\sigma}y)} - K)\phi(y)dy \end{aligned} \tag{7. 5}$$

where J is the set $[x : e^{nx} > K]$. The following computations give an explicit formula for the function $C(S(0), T)$. Observe that

(7. 6)

$$\begin{aligned}
C(S(0),T) &= e^{-\int_0^T r(s)ds+n\tilde{\mu}} \int_{-D_2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{n\tilde{\sigma}y-\frac{y^2}{2}}dy \\
&\quad -Ke^{-\int_0^T r(s)ds} \int_{-D_2}^{\infty} \phi(y)dy \\
&= e^{-\int_0^T r(s)ds+n\tilde{\mu}+\frac{1}{2}n^2\tilde{\sigma}^2} \int_{-D_2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-n\tilde{\sigma})^2}dy - Ke^{-\int_0^T r(s)ds}\Phi(D_2) \\
&= \exp\{-\int_0^T r(s)ds + n\tilde{\mu} + \frac{1}{2}n^2\tilde{\sigma}^2\}\Phi(D_1) - Ke^{-\int_0^T r(s)ds}\Phi(D_2) \\
&= S^n(0)\exp\{-\int_0^T r(s)ds + \frac{n}{T}\int_0^T\int_0^t (r(s)-q(s))dsdt \\
&\quad -\frac{n}{4}\sigma^2 T + \frac{n^2}{6}\sigma^2 T - \frac{n}{2(2H+1)}\sigma^2 T^{2H} + \frac{n^2}{4(H+1)}\sigma^2 T^{2H}\}\Phi(D_1) \\
&\quad -Ke^{-\int_0^T r(s)ds}\Phi(D_2)
\end{aligned}$$

where D_1 and D_2 are given by

(7. 7)

$$\begin{aligned}
\log D_2 &= \log[\log(S(0)/K^{1/n}) + \frac{1}{T}\int_0^T\int_0^t (r(s)-q(s))dsdt - \frac{1}{4}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H}] \\
&\quad -\log[\sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}]
\end{aligned}$$

and

(7. 8)
$$D_1 = D_2 + \sqrt{\frac{n^2\sigma^2 T}{3} + \frac{n^2\sigma^2 T^{2H}}{2(H+1)}}.$$

By similar arguments, it follows that the price of Asian Power put option is given by

(7. 9)

$$\begin{aligned}
P(S(0),T) &= Ke^{-\int_0^T r(s)ds}\Phi(-D_2) \\
&\quad -S^n(0)\exp\{-\int_0^T r(s)ds + \frac{n}{T}\int_0^T\int_0^t (r(s)-q(s))dsdt \\
&\quad -\frac{n}{4}\sigma^2 T + \frac{n^2}{6}\sigma^2 T - \frac{n}{2(2H+1)}\sigma^2 T^{2H} + \frac{n^2}{4(H+1)}\sigma^2 T^{2H}\}\Phi(-D_1).
\end{aligned}$$

It is now easy to see that the put-call option parity formula for Asian power options with constant interest rate r and constant dividend rate q is given by

(7. 10)

$$\begin{aligned} C(S(0),T)-P(S(0),T) &= S^n(0)\exp\{-rT+\frac{n}{2}(r-q)T-\frac{n}{4}\sigma^2T+\frac{n^2}{6}\sigma^2T \\ &\quad -\frac{n}{2(2H+1)}\sigma^2T^{2H}+\frac{n^2}{4(H+1)}\sigma^2T^{2H}\} \\ &\quad -Ke^{-rT} \end{aligned}$$

and the put-call option parity formula for Asian power options with non-constant interest rate $r(t)$ and dividend rate $q(t)$ is given by

(7. 11)

$$\begin{aligned} C(S(0),T)-P(S(0),T) &= S^n(0)\exp\{-\int_0^T r(s)ds+\frac{n}{T}\int_0^T\int_0^t (r(s)-q(s))dsdt \\ &\quad -\frac{n}{4}\sigma^2T+\frac{n^2}{6}\sigma^2T-\frac{n}{2(2H+1)}\sigma^2T^{2H}+\frac{n^2}{4(H+1)}\sigma^2T^{2H}\} \\ &\quad -K\exp\{-\int_0^T r(s)ds\}. \end{aligned}$$

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References :

Bjork, T. and Hult, H. (2005) A note on Wick products and the fractional Black-Scholes model, *Finance Stoch.*, **9**, 197-209.

Cheridito, P. (2000) *Regularizing fractional Brownian motion with a view toward stock price modeling*, Ph.D. Dissertation, ETH, Zurich.

Cheridito, P. (2001) Mixed fractional Brownian motion, *Bernoulli*, **7**, 913-934.

Cheridito, P. (2003) Arbitrage in fractional Brownian motion models, *Finance Stoch.*, **7**, 533-553.

Foad, Shokorollahi and Adem, Kilicman. (2014) Pricing currency option in a mixed fractional Brownian motion with jumps environment, *Mathematical Problems in Engineering*, Volume 2014, Article ID 858210, 13 pp., Hindawi Publishing Corporation, http://dx.doi.org/10.1155/2014/858210.

Kuznetsov, Yu. (1999) The absence of arbitrage in a model with fractal Brownian motion, *Russian Math. Surveys*, **54**, 847-848.

Mao, Zhijuan and Liang, Zhian. (2014) Evaluation of geometric Asian options under fractional Brownian motion, *Journal of Mathematical Economics*, **4**, 1-9.

Mishura, Yulia (2008) *Stochastic Calculus for Fractional Brownian Motion and Related Processes*, Lecture Notes in Mathematics, No. 1929, Springer, Berlin.

Prakasa Rao, B.L.S. (2010) *Statistical Inference for Fractional diffusion Processes*, Wiley, Chichester, UK.

Prakasa Rao , B.L.S. (2013) *Introduction to Statistics in Finance*, Lecture Notes, CR Rao Advanced Institute of Mathematics, Statistics and Computer Science, Hyderabad, India, pp. 156.

Prakasa Rao, B.L.S. (2015) Option pricing for processes driven by mixed fractional Brownian motion with superimposed jumps, *Probab. Engg. Info. Sci.* doi:10.1017/S0269964815000200.

Ross, Sheldon M. (2003) *An Elementary Introduction to Mathematical Finance*, Second Ed., Cambridge University Press, Cambridge, UK.

Shreve, S.E. (2004) *Stochastic Calculus for Finance II Continuous-Time Models*, Springer, New York.

Sun, L. (2013) Pricing currency options in the mixed fractional Brownian motion, *Physica A*, **392**, 3441-3458.

Sun, Xichao and Yan, Litan (2012) Mixed-fractional models in credit risk pricing, *Journal of Statistical and Econometric Methods*, **1**, 79-96.

Xiao, W.L., Zhang, W.G., Zhang, X.L., and Wang, Y.L. (2012) Pricing model for equity warrants in a mixed fractional Brownian environment and its algorithm, *Physica A*, **391**, 6418-6431.

Yu, Z. and Yan, L. (2008) European call option pricing under a mixed fractional Brownian motion environment, *J. University of Science and Technology of Suzhou*, **25**, 4-10 (in Chinese).

Zili, M. (2006) On the mixed fractional Brownian motion, *J. Appl. Math. Stochastic Anal.*, DOI10.1155/JAMSA/2006/32435.